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SHORT NOTES

Stretch in shear zones: implications for section balancing

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Abstract—The stretch, S, of a line in a zone of general shear deformation is given by $S = W \sin\theta/\sin\theta'$, where the shear zone's final to initial width ratio is W and the line's inclination to the zone boundary is θ before deformation and θ' after. For the special cases of simple shear and pure shear, $S = \sin\theta/\sin\theta'$, and $S^2 = \sin 2\theta/\sin 2\theta'$, respectively. Fisher's and Sorby's formulae may be combined in a form appropriate to general shear strain, $\gamma = \cot\theta' - R\cot\theta$, where R is the axial ratio of the irrotational component of deformation and γ is the shear strain; for simple shear and pure shear the equation reduces to $\gamma = \cot\theta' - \cot\theta$, and $\tan\theta = R\tan\theta'$, respectively. These relationships may be used to estimate strain in exposed faulted strata, or to restrict the possible geometries of inferred fault ramps in balanced geological cross-sections.

RAMSAY & HUBER (1983, equations 1.4 & 1.7) give expressions for the longitudinal strain of a line in a zone of homogeneous simple shear. In simplified form, they are

$$S^2 = 1 - \gamma \sin 2\theta + \gamma^2 \sin^2\theta \quad (1)$$

$$S^{-2} = 1 + \gamma \sin 2\theta' + \gamma^2 \sin^2\theta', \quad (2)$$

where S is the line's stretch, θ and θ' are its initial and final inclinations to the zone boundary, and γ is the simple shear strain. These formulae are useful for modelling theoretical zones of predetermined simple shear strain, but they are not easily solved for γ, given the stretch and orientation of a linear marker. Furthermore, the assumption of simple shear is too restrictive as it excludes most natural strains which comprise pure and simple components. Even three-dimensional simple shear zones display changes in width in a two-dimensional section other than a principal or circular one (Skjernaa 1980). I have therefore derived more general expressions for stretch in shear zones. The following treatment is two-dimensional, but does not assume plane strain.

Let L be the length of a line traversing a shear zone of unit width at an angle θ (Fig. 1). Clearly, $\sin\theta = 1/L$. After general shear deformation, let the zone's width be W, the line's length L', and its orientation θ', so that $\sin\theta' = W/L'$. The stretch $S\,(=L'/L)$ is therefore given by

$$S = W \sin\theta/\sin\theta'. \quad (3)$$

For the special case of simple shear ($W = 1$),

$$S = \sin\theta/\sin\theta'. \quad (4)$$

For the special case of pure shear (axial ratio $R = 1/W^2$), Sorby's formula ($R = \tan\theta/\tan\theta'$) gives

$$W^2 = \tan\theta'/\tan\theta. \quad (5)$$

Substituting for W^2 in equation (3) squared and converting to double angles,

$$S^2 = \sin 2\theta/\sin 2\theta'. \quad (6)$$

All of the above variables are illustrated on off-axis Mohr circles for stretch in Fig. 2. Equation (4) permits one to determine the initial orientation θ of a line from its observed stretch S and final orientation θ', assuming constant zone width ($W = 1$). Alternatively, if θ is measured outside the shear zone, W may be calculated in equation (3). It is not generally possible to calculate the axial ratio R of the zone-parallel irrotational component of deformation. However, it may be valid to assume volume loss normal to the zone boundary ($R = 1/W$) or pure shear ($R = 1/W^2$). Expanding Fisher's formula to the general shear case,

$$\gamma = \cot\theta' - R\cot\theta. \quad (7)$$

The shear strain is determined by adopting either of the

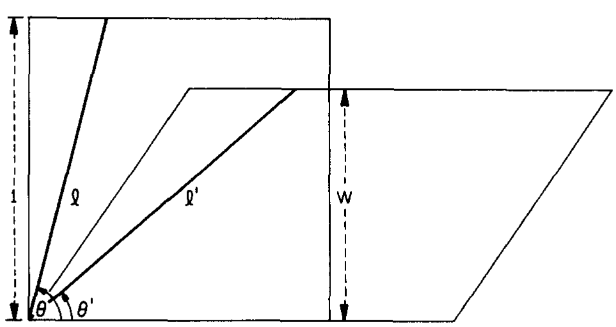

Fig. 1. An initial unit square is traversed by a line of length L and orientation θ. After deformation L', θ' and W are line length, orientation and zone width. Area not necessarily conserved.

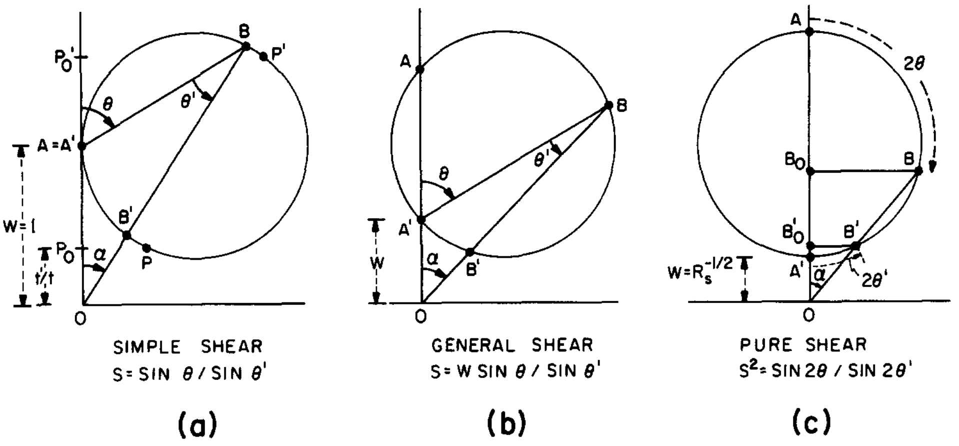

Fig. 2. Mohr circles for stretch. Area conserving cases chosen so one circle represents forward and reciprocal stretch if different sign conventions are used for θ and θ'. A,A' = eigenvectors. Polar co-ordinates represent stretch (S = OB = 1/OB') and rotation (α = AOB = A'OB'). Initial orientation = 1/2 arc AB, final = 1/2 arc A'B'. Final/initial zone width W = OA' (=1/OA). Formulae derived from rule of sines in triangles OA'B in cases (a) and (b), and from similar triangles in case (c). Pole to initial bedding = P, final = P'. For case (a) only, thickness ratio t'/t = OP_0 (=1/OP_0'). Mohr circles of the Second Kind (De Paor & Means 1984).

above assumptions for R and then eliminating W using equation (3). For volume loss,

$$\gamma = \cot\theta' - \cot\theta / W \tag{8}$$

(compare Ramsay 1980, equation 21). For a pure shear component ($R = 1/W^2$), equation (3) yields

$$R = \sin^2\theta / S^2 \sin^2\theta'. \tag{9}$$

Thus, both γ and R can be determined from θ, θ' and S. In the special cases of pure shear without simple shear ($\gamma = 0$) and simple shear without pure shear ($R = 1$), equation (7) reduces to Sorby's and Fisher's formula, respectively.

Equations (3), (4), (6) and (7) are so simple and fundamental, one wonders why they were not derived in the last century. An analogy of the simple shear case has appeared in Thompson (1960) in the context of fault block tilting (see also Albrecht 1966, Skjernaa 1980, Ragan 1985), and the inverse problem of determining thickness changes (from t to t') of dykes traversing simple shear zones was solved by Escher *et al.* (1975; also Ramsay 1980, p. 92). Being based on the rule of sines, their formula

$$t'/t = \sin\theta' / \sin\theta \tag{10}$$

looks like equation (4), but note that θ and θ' are

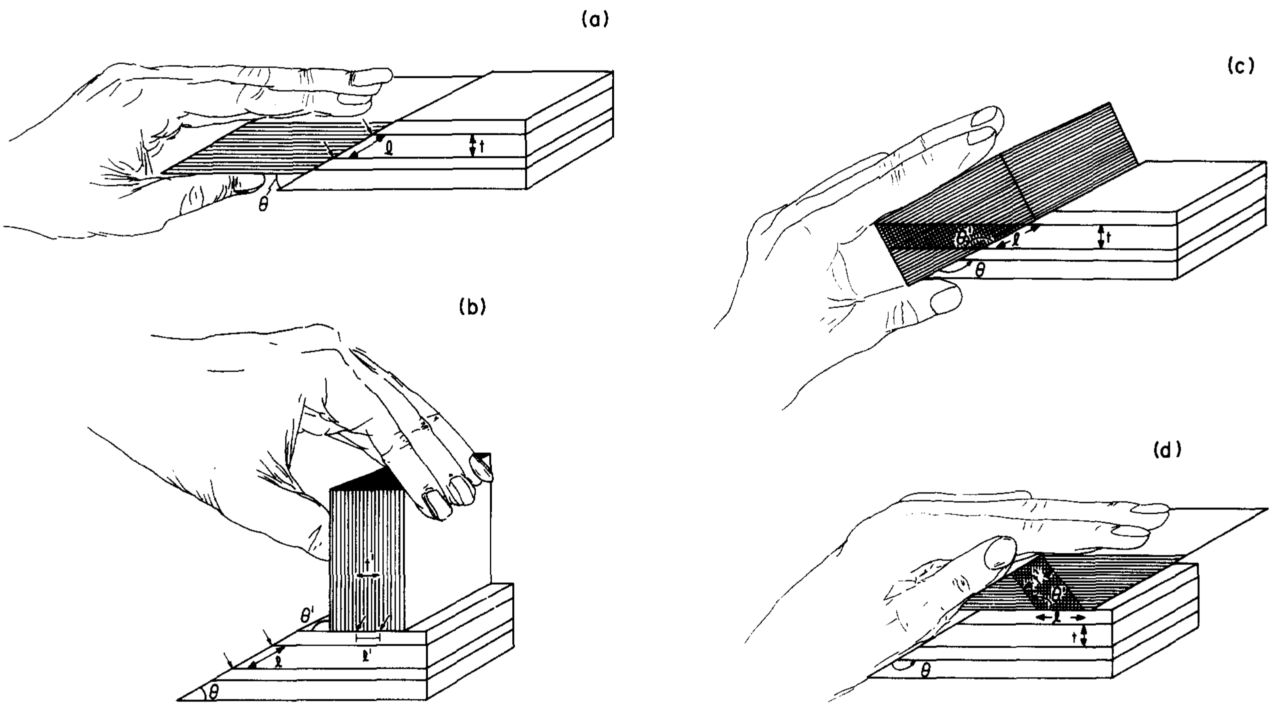

Fig. 3. Applications to section balancing. Footwall represented by wooden block. (a) Hold card deck against fault, slide to fit ramp angle, mark ramp length and bed thickness. (b) Move deck over fault bend, add/delete cards and shear as shown. (c) Hold cards with face against ramp and trace bed on side of deck. (d) Move deck over fault bend and adjust as in (b); note that L = constant, but t, θ change to t', θ'. Symbols as in Figs. 1 and 2.

interchanged and that t'/t is not a measure of stretch because the normal to the dyke margin is generally sheared and does not coincide with the same material line before and after deformation (Schwerdtner 1978). Jamison (1987) uses equation (10) to calculate thrust related fold geometries, and Suppe (1983, equation 16) employs an analogous measure for displacement changes across fault bends. However, it appears that the general equations [(3) and (7)] have not been formally presented before, and they are certainly not well known.

To examine the implications for section balancing, consider the effects of thrusting on strata as illustrated by Fig. 3. Choosing a reference frame parallel and perpendicular to bedding, $W = t'/t$. If the ramp length between two bedding cut-offs is L and the cut-off angle is θ in the footwall, then the equivalent hangingwall parameters, L' and θ', are not independent variables but are related by

$$L'/L = [t'/t][\sin \theta / \sin \theta']. \tag{11}$$

A card deck may be used to demonstrate the interdependence of variables. If there is no internal strain (cards held without sliding), then $\theta' = \theta$. Unless there are thickness changes in strata due to growth faulting, $t' = t$, implying $L' = L$. This simplest case has been proposed by Crane (1987) (see also Fischer & Coward 1982) as a rule-of-thumb for section construction. If $\theta' = \theta$, there is no internal shear (unless bedding and fault traces are both parallel to deformation eigenvectors—a contrived case); the only possible strain would be isotropic, but that could not be confined within straight zone boundaries. For constant thickness ($t' = t$), the interdependence of L' and θ' is demonstrated by moving the card deck up the fault (Fig. 3a & b) and shearing it to and fro. Thickness changes are simulated by addition or deletion of cards. Finally, if $L' = L$, equation (11) reduces to equation (10). The deformation may look general in the bedding reference frame but, if area is conserved, it is equivalent to a simple shear parallel to the fault trace, and may be simulated with the cards held facing the fault plane (Fig. 3c & d). In response to simple shear, bed thicknesses and cut-off angles satisfy Escher *et al.*'s equation [equation (10)].

In conclusion, it should be clear that the general shear deformation of hangingwall relative to footwall is entirely due to movement of strata over the fault bend. This is a spatially controlled strain which is additional to any material strain that occurred before or after fault displacement; it does not matter whether the footwall is undeformed or not (c.f. Fischer & Coward 1982). The strain recorded by classical markers such as fossils or pebbles may be factorized into components due to faulting and other causes. As far as the former is concerned, ramp lengths, bed thicknesses, and cut-off angles serve as strain markers just as if the fault had fortuitously cut straight through a trilobite! This should be encouraging to geologists who bemoan the absence of strain markers where they are most needed.

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